Renormalizability of the dimension two gluon operator A^2 in a class of nonlinear covariant gauges

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Abstract

In this work we discuss a class of nonlinear covariant gauges for Yang-Mills theories which enjoy the property of being multiplicatively renormalizable to all orders. This property follows from the validity of a linearly broken identity, known as the ghost Ward identity. Furthermore, thanks to this identity, it turns out that the local composite dimension two gluon operator $A^a_\mu A^a_\mu$ can be introduced in a mulptiplicatively renormalizable way.

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1 Introduction

Consistent perturbative quantization of gauge theories requires a choice of the gauge fixing to get rid of spurious degrees of freedom. This choice does not affect physical quantities since they correspond to gauge invariant operators. After choosing a gauge, one has to prove the renormalizability of the theory, in order to consistently define it at the quantum level. Moreover, in addition to the $BRST$ invariance, and depending on the choice of the gauge fixing, the resulting theory might display further global symmetries which reduce the number of free parameters present in the gauge fixing term.

In this letter we present a covariant nonlinear gauge fixing which enjoys the property of being multiplicative renormalizable, while containing a unique gauge parameter α. This feature stems form the existence of an additional global symmetry amounting to perform a constant shift of the Faddeev-Popov ghost field c, provided there is a compensating transformation of the Lagrange multiplier field b. This additional invariance gives rise to a linearly broken identity, known as the ghost Ward identity [1], which ensures the renormalizability of the theory. An interesting feature of this nonlinear gauge is that it reduces to the Landau gauge in the limit of vanishing gauge parameter, $\alpha \to 0$, while allowing for the introduction of the local dimension two operator $A^a_\mu A^a_\mu$, which turns out to be multiplicatively renormalizable too. This provides an example of a nonlinear gauge which allows for the introduction of the operator $A^a_\mu A^a_\mu$, which has attracted much attention in recent years.

The work is organized as follows. In section 2 we introduce the nonlinear gauge fixing, and we derive the set of Ward identities. In section 3 we prove the renormalizability of the model within the framework of the algebraic renormalization [2]. In section 4 we consider the inclusion of the dimension two composite gluon operator $A^a_\mu A^a_\mu$ and we establish its multiplicative renormalizability. Finally, the conclusions are displayed in section 5.

2 Gauge fixing

The Yang-Mills action in four-dimensional Euclidean space-time is

$$S_{YM} = \frac{1}{4}\int d^4x F^a_{\mu\nu} F^a_{\mu\nu} \ , \tag{1}$$

with the field strength given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \ , \tag{2}$$

where g is the coupling constant and f^{abc} are the structure constants of $SU(N)$, the color index a belongs to the adjoint representation, $a = 1, \ldots, N^2 - 1$.

To quantize the action (1), we follow the $BRST$ procedure and we introduce the Faddeev-Popov ghost and antighost fields, respectively, c^a and $\bar{c}^a$, as well as the Lagrange multiplier b^a.

We require invariance of the gauge fixed action under the nilpotent $BRST$ transformations

$$
\begin{aligned}
sA_{\mu}^{a} &= -D_{\mu}^{ab}c^{b} \; , \\
sc^{a} &= \frac{g}{2}f^{abc}c^{c}c^{c} \; , \\
s\bar{c}^{a} &= b^{a} \; , \\
sb^{a} &= 0 \; ,
\end{aligned} \tag{3}
$$

where the covariant derivative is defined as

$$
D_{\mu}^{ab} = \delta^{ab}\partial_{\mu} - gf^{abc}A_{\mu}^{c} \; . \tag{4}
$$

We also impose that the gauge fixed action,

$$
S = S_{YM} + S_{gf} \; , \tag{5}
$$

obeys the integrated ghost equation

$$
\mathcal{G}^{a}S = \int d^{4}x \left(\frac{\delta S}{\delta c^{a}} + gf^{abc}\bar{c}^{b}\frac{\delta S}{\delta b^{c}} \right) = 0 \; , \tag{6}
$$

which expresses in a functional form the existence of an additional global invariance, corresponding to a shift of the ghost field by a constant together with the compensating transformation of the Lagrange multiplier b^{a} [1]. Thus, the most general gauge fixing term compatible with both $BRST$ symmetry (3) and ghost equation (6) is found to be

$$
\begin{aligned}
S_{gf} &= s\int d^{4}x \; \bar{c}^{a}\left(\partial_{\mu}A_{\mu}^{a} + \frac{\alpha}{2}b^{a} + \frac{\alpha}{2}gf^{abc}\bar{c}^{b}c^{c} \right) \\
&= \int d^{4}x \left[b^{a}\left(\partial_{\mu}A_{\mu}^{a} + \frac{\alpha}{2}b^{a} + \alpha gf^{abc}\bar{c}^{b}c^{c} \right) + \bar{c}^{a}\partial_{\mu}D_{\mu}^{ab}c^{b} + \frac{\alpha}{4}g^{2}f^{abc}f^{cde}\bar{c}^{a}\bar{c}^{b}c^{d}c^{e} \right] \; .
\end{aligned} \tag{7}
$$

In order to write down the Ward identities fulfilled by the gauge fixed action, we introduce two external sources, Ω_{μ}^{a} and L^{a}, coupled to the nonlinear $BRST$ transformations [2]. Thus, for the complete starting action Σ we get

$$
\Sigma = S_{YM} + S_{gf} + S_{ext} \; , \tag{8}
$$

where

$$
\begin{aligned}
S_{ext} &= s\int d^{4}x \left(-\Omega_{\mu}^{a}A_{\mu}^{a} + L^{a}c^{a} \right) \\
&= \int d^{4}x \left(-\Omega_{\mu}^{a}D_{\mu}^{ab}c^{b} + \frac{g}{2}f^{abc}L^{a}c^{b}c^{c} \right) \; .
\end{aligned} \tag{9}
$$

As it is easily checked, the action Σ obeys the following Ward identities:

- the Slavnov-Taylor identity

$$
\mathcal{S}(\Sigma) = \int d^{4}x \left(\frac{\delta\Sigma}{\delta\Omega_{\mu}^{a}}\frac{\delta\Sigma}{\delta A_{\mu}^{a}} + \frac{\delta\Sigma}{\delta L^{a}}\frac{\delta\Sigma}{\delta c^{a}} + b^{a}\frac{\delta\Sigma}{\delta\bar{c}^{a}} \right) = 0 \; , \tag{10}
$$

fields / sources	A	c	$\bar{c}$	b	Ω	L
dimension	1	0	2	2	3	4
ghost number	0	1	-1	0	-1	-2

Table 1: Dimension and ghost number of the fields and sources.

- the linearly broken integrated ghost Ward identity

$$\mathcal{G}^a \Sigma = \Delta^a_{cl} \ , \tag{11}$$

where

$$\Delta^a_{cl} = g f^{abc} \int d^4x \left(\Omega^b_\mu A^c_\mu - L^b c^c \right) \ , \tag{12}$$

is a classical breaking [1] linear in the fields and $\mathcal{G}^a$ is given by (6).
For further use, the quantum numbers of the fields and sources are displayed in table 1.

3 Renormalizability of the model

Let us discuss now the renormalizability of the action (8). Following the framework of the algebraic renormalization [2], we shall look at the most general invariant local counterterm Σ^c compatible with the Ward identities characterizing the model. From equations (10) and (11) it follows thus that Σ^c fulfills the following constraints

$$\mathcal{B}_\Sigma \Sigma^c = 0 \ , \tag{13}$$

$$\mathcal{G}^a \Sigma^c = 0 \ , \tag{14}$$

where $\mathcal{B}_\Sigma$ is the nilpotent linearized Slavnov-Taylor operator

$$\mathcal{B}_\Sigma = \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + b^a \frac{\delta}{\delta\bar{c}^a} \right) \ . \tag{15}$$

From the general results on the cohomology of Yang-Mills theories, see [2] and references therein, it follows that the most general solution of eq.(13) can be written as

$$\Sigma^c = a_0 S_{YM} + \mathcal{B}_\Sigma \Delta^{-1} \ , \tag{16}$$

where Δ^{-1} is an integrated polynomial in the fields and sources, with dimensions bounded by four and negative ghost number, namely

$$\Delta^{-1} = \int d^4x \left(a_1 \partial_\mu \bar{c}^a A^a_\mu + a_2 L^a c^a + a_3 \frac{\alpha}{2} \bar{c}^a b^a + a_4 \frac{\alpha}{2} g f^{abc} \bar{c}^a \bar{c}^b c^c + a_5 \Omega^a_\mu A^a_\mu \right) \ , \tag{17}$$

where a_0, a_1, a_2, a_3, a_4, a_5 are free coefficients. Furthermore, from the eq.(14), it follows that $a_4 = -2a_3$ and $a_2 = 0$. Thus, the most general invariant counterterm turns out to contains four free parameters, a_0, a_1, a_3, a_5, being given by

$$\Sigma^c = a_0 S_{YM} + \mathcal{B}_\Sigma \int d^4x \left[a_1 \partial_\mu \bar{c}^a A^a_\mu + a_3 \frac{\alpha}{2} \bar{c}^a \left(b^a - 2g f^{abc} \bar{c}^b c^c \right) + a_5 \Omega^a_\mu A^a_\mu \right] \ . \tag{18}$$

After having characterized the most general counterterm, it remains to check the stability of the action (8), amounting to prove that the counterterm Σ^c can be reabsorbed by a multiplicative redefinition of the parameters, fields, and sources of Σ, according to

$$\Sigma(\Phi, J, \xi) + \epsilon\Sigma^c(\Phi, J, \xi) \ = \Sigma(\Phi_0, J_0, \xi_0) + O(\epsilon) \ , \tag{19}$$

with

$$\begin{array}{rclcrcl} \Phi_0 & = & Z_\Phi^{1/2}\Phi \ , & & \Phi & \in & \{A, c, \bar{c}\} \ , \\ J_0 & = & Z_J J \ , & & J & \in & \{\Omega, L\} \ , \\ \xi_0 & = & Z_\xi \xi \ , & & \xi & \in & \{g, \alpha\} \ , \end{array} \tag{20}$$

In fact, by direct inspection one finds

$$\begin{array}{rcl} Z_A^{1/2} & = & 1 + \epsilon\left(\dfrac{a_0}{2} + a_5\right) \ , \\ Z_c^{1/2} = Z_{\bar{c}}^{1/2} & = & 1 - \epsilon\dfrac{a_1}{2} \ , \\ Z_g & = & 1 - \epsilon\dfrac{a_0}{2} \ , \\ Z_\alpha & = & 1 + \epsilon\left(a_0 + 2a_1 + a_3\right) \ , \end{array} \tag{21}$$

and

$$\begin{array}{rcl} Z_b^{1/2} & = & 1 - \epsilon\left(\dfrac{a_0}{2} + a_1\right) \ = \ Z_g Z_c \ , \\ Z_\Omega & = & Z_g^{-1} Z_A^{-1/2} Z_c^{-1/2} \ , \\ Z_L & = & Z_g^{-1} Z_c^{-1} \ , \end{array} \tag{22}$$

thus establishing the multiplicative renormalizability of the action Σ.

4 Inclusion of the dimension two gluon operator

Let us discuss now the inclusion of the dimension two gluon operator $A^a_\mu A^a_\mu$ in the case of the nonlinear gauge (7). According to [3, 4, 5], we add the operator $A^a_\mu A^a_\mu$ to the action (8) through the following term

$$\begin{array}{rcl} S_{LCO} & = & s\displaystyle\int d^4x \left(\lambda\frac{A^a_\mu A^a_\mu}{2} + \frac{\zeta}{2}\lambda J\right) \ , \\ & = & \displaystyle\int d^4x \left(J\frac{A^a_\mu A^a_\mu}{2} + \lambda A^a_\mu \partial_\mu c^a + \frac{\zeta}{2}J^2\right) \ , \end{array} \tag{23}$$

where λ and J are external sources introduced as a BRST doublet

$$\begin{array}{rcl} s\lambda & = & J \ , \\ sJ & = & 0 \ . \end{array} \tag{24}$$

The quantum numbers of the external sources are displayed in table 2. The dimensionless parameter ζ is needed in order to take into account the ultraviolet divergences present in the Green function $\left\langle A^2(x)A^2(y)\right\rangle$ [3, 4, 5]. The action we will work with is now given by

$$\Sigma' = \Sigma + S_{LCO} \ , \tag{25}$$

LCO sources	λ	J
dimension	2	2
ghost number	-1	0

Table 2: Dimension and ghost number of the LCO sources.

where Σ stands for expression (8). The introduction of the operator $A^a_\mu A^a_\mu$ does not affect the $BRST$ symmetry and the ghost equation. In fact, the modified action Σ' obeys the following Slavnov-Taylor identity

$$\mathcal{S}(\Sigma') = \int d^4x \left(\frac{\delta\Sigma'}{\delta\Omega^a_\mu}\frac{\delta\Sigma'}{\delta A^a_\mu} + \frac{\delta\Sigma'}{\delta L^a}\frac{\delta\Sigma'}{\delta c^a} + b^a\frac{\delta\Sigma'}{\delta\bar{c}^a} + J\frac{\delta\Sigma'}{\delta\lambda} \right) = 0 \ , \tag{26}$$

while the ghost equation (11) remains unaffected,

$$\mathcal{G}^a\Sigma' = \Delta^a_{cl} \ , \tag{27}$$

with $\mathcal{G}^a$ and Δ^a_{cl} given, respectively, by (6) and (12).

The multiplicative renormalizability of the action (25) can be established in the same way as that of the action (8). For that, one looks at the most general invariant counterterm $\widetilde{\Sigma}$, which is an integrated polynomial in the fields and sources with dimension bounded by four and with vanishing ghost number. From the Slavnov-Taylor identity (26), it follows that $\widetilde{\Sigma}$ can be written as

$$\widetilde{\Sigma} = a_0 S_{YM} + \mathcal{B}'_\Sigma \Delta^{-1} \ , \tag{28}$$

where Δ^{-1} reads

$$\begin{aligned}\Delta^{-1} \ &= \int d^4x \left(a_1\partial_\mu\bar{c}^a A^a_\mu + a_2 L^a c^a + a_3\frac{\alpha}{2}\bar{c}^a b^a + a_4\frac{\alpha}{2}g f^{abc}\bar{c}^a\bar{c}^b c^c + a_5\Omega^a_\mu A^a_\mu \right. \\ &+ \left. a_6\frac{\lambda}{2}A^a_\mu A^a_\mu + a_7\frac{\zeta}{2}\lambda J + a_8\bar{c}^a c^a \right) \ ,\end{aligned} \tag{29}$$

and $\mathcal{B}'_\Sigma$ is the linearized nilpotent operator corresponding to the Slavnov-Taylor identity (26), namely

$$\mathcal{B}'_\Sigma = \int d^4x \left(\frac{\delta\Sigma'}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma'}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma'}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma'}{\delta c^a}\frac{\delta}{\delta L^a} + b^a\frac{\delta}{\delta\bar{c}^a} + J\frac{\delta}{\delta\lambda} \right) \ . \tag{30}$$

Moreover, the ghost identity (27) implies that $\widetilde{\Sigma}$ is constrained by

$$\mathcal{G}^a\widetilde{\Sigma} = 0 \ , \tag{31}$$

from which it follows that $a_2 = 0$, $a_4 = -2a_3$ and $a_8 = 0$. Thus, for the final form of the counterterm we get

$$\begin{aligned}\widetilde{\Sigma} \ &= a_0 S_{YM} + \mathcal{B}'_\Sigma \int d^4x \left[a_1\partial_\mu\bar{c}^a A^a_\mu + a_3\frac{\alpha}{2}\bar{c}^a\left(b^a - 2g f^{abc}\bar{c}^b c^c\right) + a_5\Omega^a_\mu A^a_\mu \right. \\ &+ \left. a_6\frac{\lambda}{2}A^a_\mu A^a_\mu + a_7\frac{\zeta}{2}\lambda J \right] \ .\end{aligned} \tag{32}$$

As done in the previous section, we have to check that the counterterm $\widetilde{\Sigma}$ corresponds to a redefinition of the fields, sources and parameters of the action Σ'. In fact, it turns out that the action (25) is multiplicatively renormalizable. The renormalization of the fields, $BRST$ sources and coupling constant are given as in eqs.(20), (21) and (22). Also, the parameter ζ and the sources λ, J renormalize as

$$
\begin{aligned}
\zeta_0 &= Z_\zeta \zeta \;, \\
\lambda_0 &= Z_\lambda \lambda \;, \\
J_0 &= Z_J J \;.
\end{aligned} \tag{33}
$$

with

$$
\begin{aligned}
Z_\zeta &= 1+\epsilon\left(2a_0-2a_6+a_7\right) \;, \\
Z_\lambda &= 1+\epsilon\left(-\frac{a_0}{2}+\frac{a_1}{2}+a_6\right) \;=\; Z_g^{-1}Z_c^{-1}Z_J \;, \\
Z_J &= 1+\epsilon\left(a_6-a_0\right) \;.
\end{aligned} \tag{34}
$$

Notice, in particular, that the source J, and thus the composite operator $A^a_\mu A^a_\mu$ coupled to it, displays multiplicative renormalizability.

5 Discussion and conclusions

In this work we have discussed a class of nonlinear covariant gauges characterized by the validity of the integrated broken ghost Ward identity [1]. This identity, together with the Slavnov-Taylor identity, has enabled us to prove the multiplicative renormalizability of the theory, a feature which has been established to all orders of perturbation theory by means of the algenraic renormalization [2]. Further, we have been able to introduce the dimension two gluon operator $A^a_\mu A^a_\mu$, while maintaining the renormalizability of the model.

To some extent, the example of the covariant linear gauges presented here enlarges the number of gauges for which a local dimension two operator can be introduced in a multiplicatively renormalizable way. To our knowledge, such a dimension two operator can be, in practice, introduced in almost all known renormalizable covariant gauges, namely: the Landau gauge [3, 4, 5], the linear covariant gauges [6], the Curci-Ferrari gauge [7], the maximal Abelian gauge* [8, 9], as well as in a variety of interpolating gauges [9, 10].

Although the operator $A^a_\mu A^a_\mu$ is not gauge invariant, the property of being multiplicatively renormalizable in a rather large number of gauges can be interpreted as evidence in favor of its relevance for the infrared behavior of the gluon propagator. It is worth reminding here that the condensate $\left\langle A^a_\mu A^a_\mu \right\rangle$ is in fact directly related to the appearance of an effective dynamical gluon mass, a topic which is receiving increasing attention in recent years [11, 12, 13, 3, 14, 15, 16, 17, 18, 19, 20].

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*In the case of the Curci-Ferrari and maximal Abelian gauges a generalized dimension two operator, *i.e.* $\left(\frac{A^2}{2}+\alpha\bar{c}c\right)$, has to be considered.

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References

[1] A. Blasi, O. Piguet and S. P. Sorella, Nucl. Phys. B **356**, 154 (1991).

[2] O. Piguet and S. P. Sorella, "Algebraic renormalization: Perturbative renormalization, symmetries and Lect. Notes Phys. **M28**, 1 (1995).

[3] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, Phys. Lett. B **516** (2001) 307.

[4] R. E. Browne and J. A. Gracey, JHEP **0311** (2003) 029.

[5] D. Dudal, H. Verschelde and S. P. Sorella, Phys. Lett. B **555**, 126 (2003) [arXiv:hep-th/0212182].

[6] D. Dudal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, JHEP **0401**, 044 (2004) [arXiv:hep-th/0311194].

[7] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, Annals Phys. **308**, 62 (2003).

[8] K. I. Kondo, T. Murakami, T. Shinohara and T. Imai, Phys. Rev. D **65**, 085034 (2002) [arXiv:hep-th/0111256].

[9] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and H. Verschelde, Phys. Rev. D **70**, 114038 (2004) [arXiv:hep-th/0406132].

[10] M. A. L. Capri, R. F. Sobreiro and S. P. Sorella, Phys. Rev. D **73**, 041701 (2006) [arXiv:hep-th/0512096].

[11] M. J. Lavelle and M. Schaden, Phys. Lett. B **208**, 297 (1988).

[12] F. V. Gubarev and V. I. Zakharov, Phys. Lett. B **501**, 28 (2001) [arXiv:hep-ph/0010096].

[13] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, Phys. Rev. Lett. **86**, 2220 (2001) [arXiv:hep-ph/0010057].

[14] K. I. Kondo, Phys. Lett. B **514**, 335 (2001) [arXiv:hep-th/0105299].

[15] J. A. Gracey, Eur. Phys. J. C **39**, 61 (2005) [arXiv:hep-ph/0411169].

[16] X. d. Li and C. M. Shakin, Phys. Rev. D **71**, 074007 (2005) [arXiv:hep-ph/0410404].

[17] P. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **63**, 114003 (2001) [arXiv:hep-ph/0101302].

[18] P. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, F. De Soto, A. Donini, H. Moutare and J. Rodriguez-Quintero Phys. Rev. D **66**, 034504 (2002) [arXiv:hep-ph/0203119].

[19] P. Boucaud, F. de Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, H. Moutarde, O. Pene and J. Rodriguez-Quintero, arXiv:hep-lat/0504017.

[20] E. Ruiz Arriola, P. O. Bowman and W. Broniowski, Phys. Rev. D **70**, 097505 (2004) [arXiv:hep-ph/0408309].